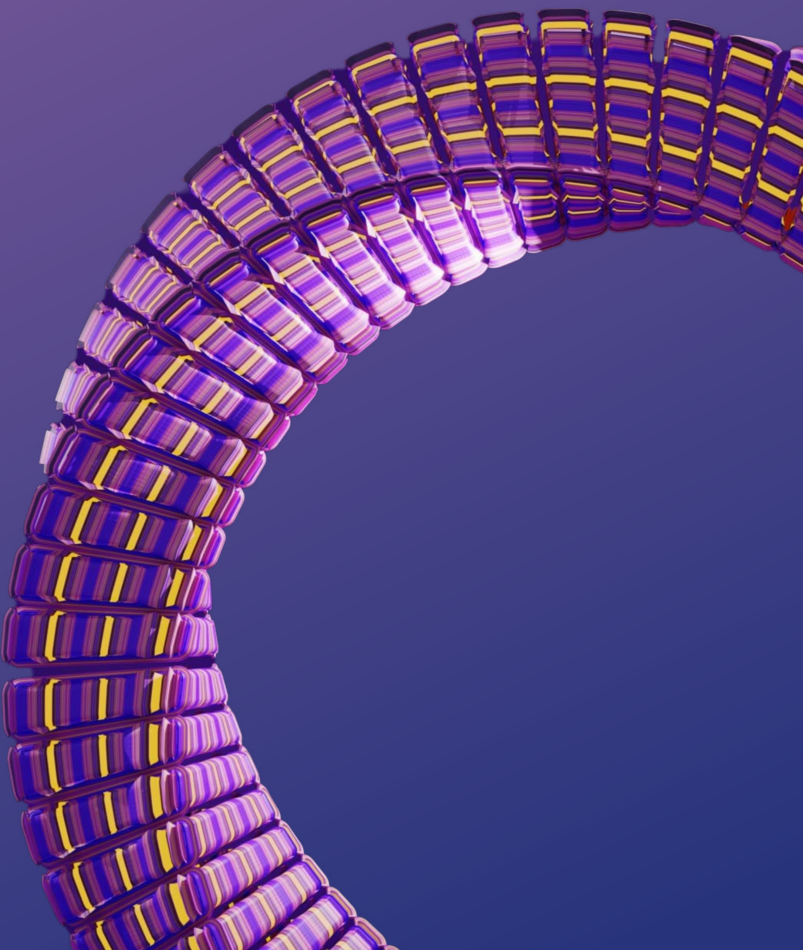
FRANCESCO CESARI - GIANGIACOMO MINAK

Machine Design for Structural Integrity



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PREFACE

This volume gathers an organic and progressive set of lecture notes and worked exercises, developed from a lifelong teaching experience. The book is intended as an operational companion to support students, in particular those of the Machine Design course delivered at the Forlì campus within the international Master's Degree programme in Mechanical Engineering for Sustainability, in the study and understanding of the main methods used to analyse, verify, and size mechanical components and structures, by integrating essential theoretical foundations with numerical applications that reflect common engineering practice.

The contents are organized along a coherent learning path. The first chapters introduce the fundamentals of three-dimensional stress and strain, elastic constitutive laws, and the governing equations of elasticity, providing the language needed to interpret mechanical response and to build reliable design checks. The text then presents the finite element method for plane problems, and on the practical use and limitations of common element types.

Subsequent chapters address classical plate bending theory and the membrane and bending behaviour of shells of revolution, with particular attention to pressure components such as cylindrical and spherical vessels and their heads. These topics provide the analytical background required to tackle thin-walled structures encountered in industrial applications, including the interpretation of stress resultants, boundary layers, and edge constraints. Where appropriate, the connection with engineering design practice and verification criteria (including stress classification and design limits for pressure equipment) is highlighted.

The second part of the book is devoted to structural integrity and durability. Elasto-plastic analysis is introduced through uniaxial and multiaxial yielding, flow and hardening rules, residual stresses, and representative analytical solutions. Linear elastic fracture mechanics is then developed through stress intensity factors, toughness, fatigue-induced crack growth, and elastic-plastic concepts such as crack-tip plasticity. The discussion is complemented by a section on low-cycle fatigue, covering cyclic stress–strain behaviour, the Coffin–Manson framework, and common correction rules for notched components. Two final chapters are dedicated to creep and to the viscoelastic behaviour of polymers, providing the basis for treating time-dependent deformation and its implications for the design and verification of plastic, composite parts, and metallic components when operating at elevated temperatures.

Throughout the volume, the emphasis is not only on obtaining numerical results but also on developing sound engineering judgement: how to set up a problem correctly, choose appropriate models, understand the meaning of the quantities involved, and critically assess the assumptions behind the calculations. Exercises and worked examples—ranging from recurring application cases to past exam-style problems—are presented in increasing order of difficulty to foster a rigorous and methodical approach.

The breadth and depth of the topics make the material suitable as a reference and exercise book for analogous courses in master's degree programmes in Nuclear Engineering and Chemical Engineering, where pressure equipment, structural integrity, fracture, fatigue, and time-dependent phenomena are equally central.

We are grateful to the students whose feedback over the years contributed to improving both the clarity of the explanations and the selection of the exercises. Any remaining errors are our responsibility, and we welcome suggestions for future editions.

April 2026 Giangiacomo Minak and Francesco Cesari

1.
THE THEORY OF ELASTICITY

The approximations underlying the elementary theory of structural engineering do not allow for the resolution of most engineering problems that cannot be modeled with a beam or frame model. The purpose of elasticity theory (TE) is to provide a mathematical model capable of describing and predicting the mechanical behavior of deformable bodies subjected to external forces, constraints, assuming that the material operates within the elastic range. In particular, elasticity theory aims to:

- determine the displacement field within the body
- derive the strain field from the displacements
- compute the corresponding stress field
- satisfy the equilibrium equations and the boundary conditions

The theory is based on three fundamental pillars:

1. equilibrium equations, ensuring force and moment balance
2. kinematic relations, linking displacements and strains
3. constitutive laws, relating stresses and strains

Overall, elasticity theory forms the foundation of structural analysis, solid mechanics, the finite element method, and many applications in mechanical and civil engineering.
In this chapter, we demonstrate the exact solution of some simple structures collected in the famous text by *Timoshenko-Goodier: Theory of Elasticity.*
From a mathematical standpoint, elementary structural mechanics-such as beam, bar, and frame theory-reduces the mechanical problem to ordinary differential equations (ODE), since the unknown fields depend on a single spatial variable along the structural axis. In contrast, three-dimensional elasticity theory describes the full stress and displacement fields within a continuum body.
The unknowns depend on several spatial coordinates, and the governing equilibrium, compatibility, and constitutive relations lead to systems of partial differential equations (PDE). This distinction reflects the different levels of modeling: structural theories introduce simplifying kinematic assumptions that reduce the dimensionality of the problem, whereas elasticity theory provides a more general and rigorous continuum formulation.

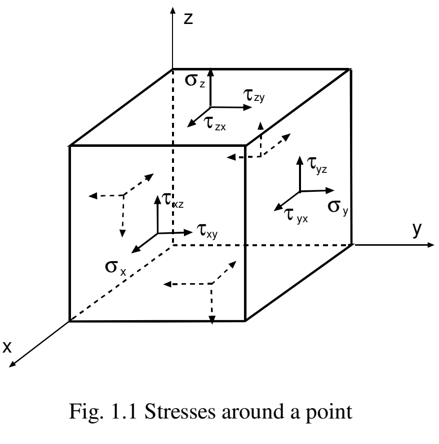

Fig. 1.1 Stresses around a point

1.1 STRESSES AROUND A POINT

Let us consider the elementary rectangular parallelepiped in Fig. 1.1 with sides parallel to the coordinate axes and the corresponding state of stress. Normal stresses are distinguished by a single subscript, which represents the axis to which the face is normal and along which the stress acts. Normal stress is positive if tensile and negative if compressive. The shear stress on each face, defined by two components parallel to the coordinate axes, requires two subscripts, the first of which indicates the normal to the plane on which the stress acts and the second its direction. A shear stress is positive if it acts on a positive face in the positive direction or if it acts on a negative face in the negative direction. Following this rule, the positive stresses shown in the positive part of figure all have the direction of the coordinate axes, while the positive stresses on the negative part are reversed. In many practical engineering problems, one geometric dimension (usually the thickness) plays a special role. Depending on its size and on how deformation is constrained, the three-dimensional problem can be reduced to two dimensions.

Plane stress: the third dimension is very small, the body is very thin in the thickness direction (z-direction) and the surfaces normal to z are free (no traction applied).
Plane strain: the third dimension is very large, the body is very thick or very long in the z-direction and geometry, loads, and material properties do not vary along z.

1.1.1 Plane stress

Consider a structure where:

- the length in one dimension is much smaller than the other two ($L_z << L_x, L_y$)
- loads are applied only within a plane
- the in-plane stresses, strains and displacements are constant in the third dimension
- normal and shear stresses in the third dimension are negligible.

Such a structure is said to be in a state of plane stress (Fig. 1.2).
The stresses σ_z, τ_{xz}, τ_{yz} are zero on both faces of the plate and approximately also inside the plate. The stresses present are σ_x, σ_y, τ_{xy} and the stress state is called the plane stress state

(1.1) $$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix}$$

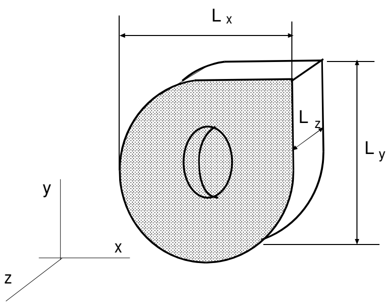

Fig. 1.2 Plane stress

1.1.2 Plane strain

Next we consider a structure where:

- the length in one dimension is much larger than the other two ($L_z << L_x$, L_y)
- loads are applied only within a plane-
- loads are constant in the third dimension
- displacements and strains along the third dimension are negligible.

Such a structure is said to be in a state of plane strain (Fig. 1.3). The stress vector is still given by (1.1), with the difference that now there is a stress σ_z that maintains a state of plane strain.

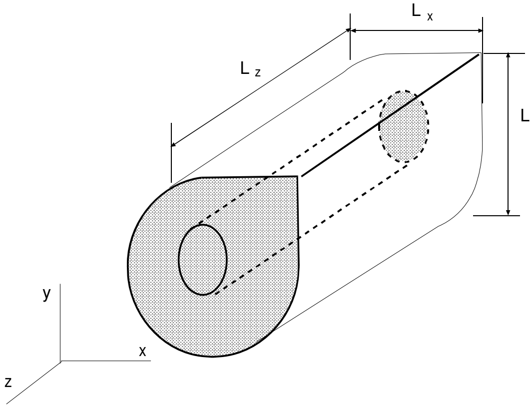

Fig. 1.3 Plane strain

1.2 EQUILIBRIUM EQUATIONS

Equilibrium equations express the condition that the body is in static equilibrium. They require that the sum of forces and the sum of moments acting on any infinitesimal element are equal to zero. These equations ensure that the stress components are distributed in such a way that the body does not undergo rigid-body motion.

The equilibrium equations are purely mechanical and are always valid, regardless of the material properties and the state of stress.

The type of structure studied is related to a plate stressed by forces applied to the edge, parallel to the plane of the plate and uniformly distributed throughout its thickness (Fig. 1.4a).

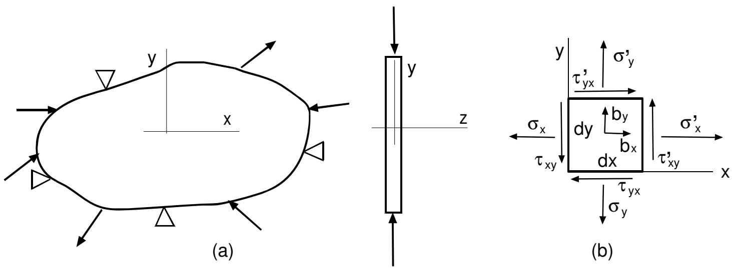

Fig. 1.4 Equilibrium state

Let's consider the element dx dy in the plane stress state where we have (Fig. 1.4b)

$$\sigma'_x = \sigma_x + \frac{\partial \sigma_x}{\partial x} dx,\ \sigma'_y = \sigma_y + \frac{\partial \sigma_y}{\partial y},\ \tau'_{xy} = \tau_{xy} + \frac{\partial \tau_{xy}}{\partial x},\ \tau'_{yx} = \tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}$$

Let b_x and b_y be the volume forces. We impose equilibrium on rotation

$$\tau_{yx}\, dx \frac{dy}{2} + \tau'_{yx}\, dx \frac{dy}{2} - \tau_{xy}\, dy \frac{dx}{2} - \tau'_{xy}\, dy \frac{dx}{2} = 0$$

Neglecting the infinitesimals of higher order we find

$$\tau_{xy} = \tau_{yx}$$

For the equilibrium translation along x we have

$$\sigma'_x\, dy - \sigma_x\, dy + \tau'_{yx}\, dx - \tau_{yx}\, dx + b_x\, dx\, dy = 0$$

Similarly in the y-direction. Therefore, relative to the stress vector (1.1) we get the three equilibrium equations

$$\begin{aligned} &\tau_{xy} = \tau_{yx} \\ (1.2)\qquad &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x = 0 \\ &\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + b_y = 0 \end{aligned}$$

The (1.2) are called *indefinite equilibrium equations,* since the boundary conditions have not yet been specified.

1.3 KINEMATIC RELATIONS

The small displacements of a deformed body are described by three displacement components u, v, and w along the coordinate directions x, y, and z, respectively. These displacements are assumed to be small quantities that vary continuously throughout the volume of the body.

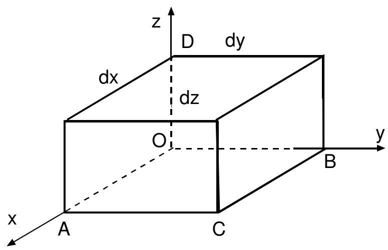

Fig. 1.5 Infinitesimal volume element

Consider an infinitesimal volume element dx dy dz of an elastic body (Fig. 1.5). If the body deforms and u, v, and w are the displacements at point O, the displacement in the x-direction of the adjacent point A along the x-axis is

$$u + \frac{\partial u}{\partial x} dx$$

due to the increment (∂u/∂x)dx of the displacement function u associated with an increment of x. The increase in the length OA caused by the deformation is therefore (∂u/∂x)dx. Hence, the unit elongation, or normal strain, at point O in the x-direction is ∂u/∂x. Similarly, in the y- and z-directions, the normal strains are ∂v/∂y and ∂w/∂z, respectively. Let us now consider the distortion of the angle between OA and OB (Fig. 1.6).

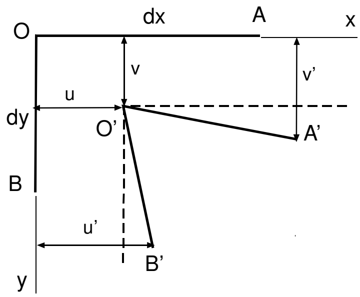

Fig. 1.6 Angular distortion of a right angle

If u and v are the displacements of point O in the x- and y-directions, respectively, the displacement of point A in the y-direction and that of point B in the x-direction are, respectively

$$v' = v + \frac{\partial v}{\partial x} dx \qquad u' = u + \frac{\partial u}{\partial y} dy$$

As a result of these displacements, the new direction O'A' of the line element OA is inclined with respect to its initial direction by a small angle equal to ∂v/∂x. Similarly, the direction O'B' is inclined with respect to OB by an angle equal to ∂u/∂y.

Therefore, the initial right angle AOB is reduced by the amount ∂v/∂x+∂u/∂y, which represents the shear strain between the xz and yz planes. In an analogous manner, the shear strains between the xy, xz, and yz planes are obtained. The unit elongation is denoted by ε, while the unit shear strain is denoted by γ.

Using the same subscripts as for the stress components, in the case of plane stress the compatibility equation is obtained

$$(1.3) \quad \boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{pmatrix}$$

Since there are three mutually perpendicular normal strains and three shear strains in the same directions, it is possible to determine the elongation in each direction and the distortion of a right angle in every plane.